

Exercises

Exercise 1 There are 3 production sites A, B, C and 5 consumption sites p_1, p_2, p_3, p_4, p_5 ; their production and consumption, respectively, are given in the following tables.

production site		A	B	C	
production		5	4	7	
consumption site	p_1	p_2	p_3	p_4	p_5
consumption	3	4	5	2	1

Finally, each production sites can only serve the consumption sites as summarized in the following table.

A	B	C
p_1, p_3	p_2, p_4	p_3, p_4, p_5

The problem is to satisfy the consumption sites. Model the following problem in terms of flows and give a solution to the problem or explain why it could not exist.

Exercise 2 Let $N = (G, s, p, c)$ be a flow network such that, for all arc e , $c(e)$ is an even integer.

1. Prove that the maximum value of a flow is an even integer.
2. Show that there is a maximum flow f such that, for all arc e , $f(e)$ is an even integer.

Exercise 3 Suppose we are in the middle of a baseball season where each team T_i , $1 \leq i \leq n$ has won $w(i)$ games so far and thus has $w(i)$ points (recall that in baseball each game has one point and we cannot have a tie.). Let G_1, G_2, \dots, G_k be the schedule of the remaining games, where each G_i is an unordered pair of teams.

Given T_i , $w(i)$, $1 \leq i \leq n$, and G_1, G_2, \dots, G_k , can we predict that T_1 does or not does not have a chance to have the top score at the end of the season? if T_1 has a chance of being champion, how can we find a sequence of outcomes (i.e. results of G_1, G_2, \dots, G_k) such that T_1 reaches the top rank at the end of the season? (of course, model this problem as a flow problem)

As an example, consider the following instance of the problem, where $g(i)$ is the number of remaining games to be played by team i , and $g(i, j)$ is the number of remaining games to be played by team i against team j . Is Harvard eliminated or not?

	$w(i)$	$g(i)$	$g(i, j)$			
Team	Wins	To play	Yale	Harvard	Cornell	Brown
Yale	33	8		1	6	1
Harvard	29	4	1		0	3
Cornell	28	7	6	0		1
Brown	27	5	1	3	1	

hint : consider the bipartite graph with vertex set $A \cup B$ where A consists of the remaining games, and B consists of each of the teams.