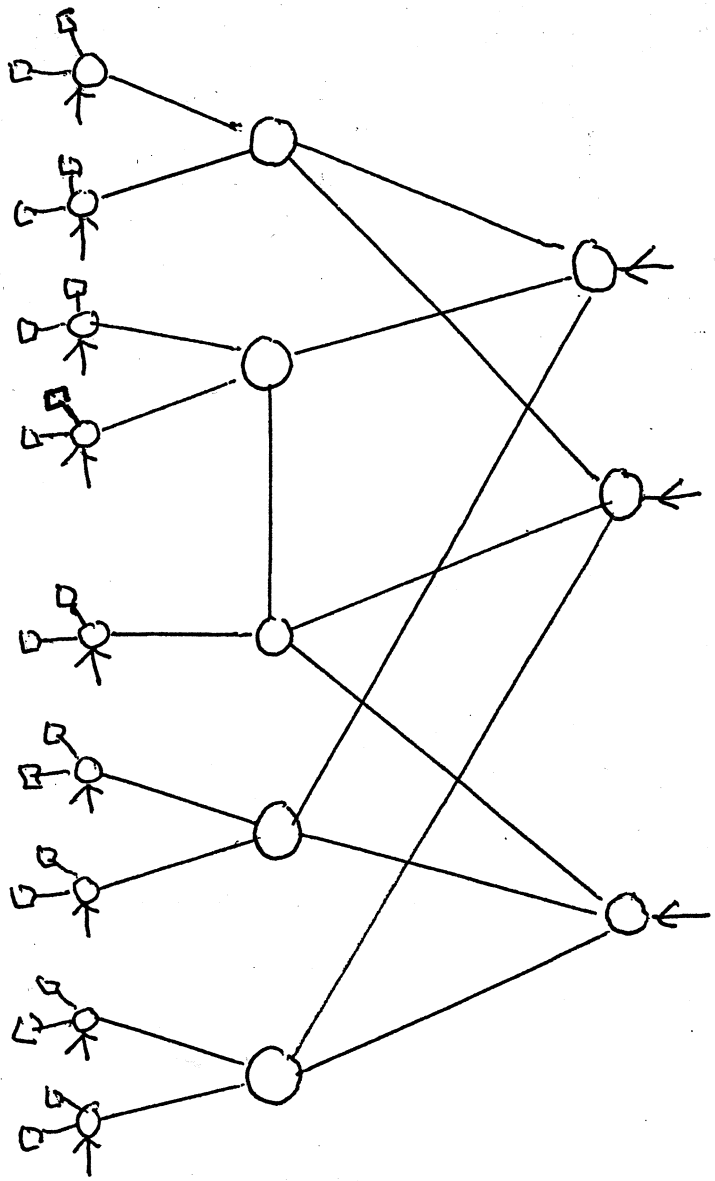
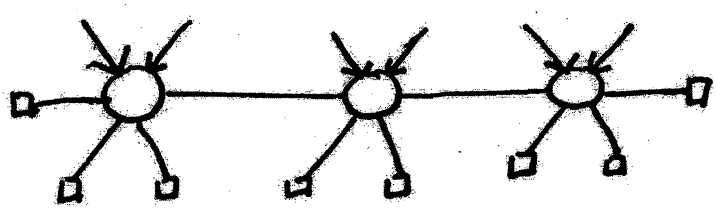


Is this network a (12,6) valid network.
(#L=12 inputs ←
12+6 amplificers ↑)

Exercice 4. (4 points) *The questions are relatively independent*

We consider (like in the Alcatel's problem considered in the first lecture ; see also the article in Networks)) a network with n inputs (signals) $n+k$ outputs (amplifiers) and valid k-fault tolerant. We suppose $k \geq 1$. But, here we suppose **each switch has 6 ports** (in the lecture and article they had only 4 ports). The objective is to design such a network with the minimum number of switches.

1) Show that the network below with $n = 6$ inputs, 8 outputs (amplifiers) tolerates $k = 2$ failures.



2) Prove that in a minimum valid (n, k)-network , with $k \geq 1$ failures , there is no switch connected to 3 or more inputs (each switch is connected to at most 2 inputs).
3) Deduce that the minimum number of switches in a valid (n, k)-network is at least $\frac{n}{2}$.
4) Design for $k = 2$ and n even a valid network with $\frac{n}{2}$ switches. Is it optimal?
5*) Design for $k = 4$ and $n = 6$ a valid $(6, 4)$-network with 4 switches.