Exercise 4. (4 points) The questions are relatively independent

We consider (like in the Alcatel's problem considered in the first lecture; see also the article in Networks) a network with $n$ inputs (signals) $n+k$ outputs (amplifiers) and valid $k$-fault tolerant. We suppose $k \geq 1$. But, here we suppose each switch has 6 ports (in the lecture and article they had only 4 ports). The objective is to design such a network with the minimum number of switches.

1) Show that the network below with $n = 6$ inputs, 8 outputs (amplifiers) tolerates $k = 2$ failures.

2) Prove that in a minimum valid $(n,k)$-network, with $k \geq 1$ failures, there is no switch connected to 3 or more inputs (each switch is connected to at most 2 inputs).

3) Deduce that the minimum number of switches in a valid $(n,k)$-network is at least $\frac{n}{2}$.

4) Design for $k = 2$ and $n$ even a valid network with $\frac{n}{2}$ switches. Is it optimal?

5*) Design for $k = 4$ and $n = 6$ a valid $(6,4)$-network with 4 switches.