Performance analysis of communication networks

IEEE 802.11 Performance Analysis

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Objectives

- Evaluate the performance of the MAC layer in IEEE 802.11 protocol

- Compute
  - transmission probability τ
  - collision probability p
  - normalized system throughput S (not station throughput)

- Side objective:

  learn about discrete-time Markov chain

Discrete-time Markov chain

- Definition

  a discrete-time Markov chain (DTMC) is a discrete-time discrete-space stochastic process X_n such that

$$P(X_{n+1} = j \mid X_0 = i_0, \ldots, X_n = i) = P(X_{n+1} = j \mid X_n = i) = p_{ij}(\cancel{n})$$

  (future values depend only on current value)

  if homogeneous

- Transition matrix

$$\mathbf{P} = \begin{pmatrix} p_{00} & p_{01} & \cdots & p_{0j} & \cdots \\ p_{10} & p_{11} & \cdots & p_{1j} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{i0} & p_{i1} & \cdots & p_{ij} & \cdots \\ \vdots & \vdots & \cdots & \vdots & \ddots \end{pmatrix}$$

$$p_{ij} \geq 0 \quad \forall i, j$$

$$\sum_j p_{ij} = 1 \quad \forall i$$

Discrete-time Markov chain

- Problem

compute the stationary distribution of the process X_n

$$\pi(i) = \lim_{n\to\infty} \pi_n(i), \quad \pi_n(i) = P(X_n = i)$$

- Bayes' formula

$$\underbrace{P(X_{n+1} = j)}_{} = \sum_i \underbrace{P(X_{n+1} = j \mid X_n = i)}_{}\underbrace{P(X_n = i)}_{}$$

$$\pi_{n+1}(j) = \sum_i p_{ij}\,\pi_n(i) \qquad \forall\, j, n$$

- let $\boldsymbol{\pi}_n = (\pi_n(0), \pi_n(1), \ldots, \pi_n(i), \ldots)$ then $\boldsymbol{\pi}_{n+1} = \boldsymbol{\pi}_n \mathbf{P}$

- Normalization condition $\sum_i \pi_n(i) = 1 \quad \Leftrightarrow \quad \boldsymbol{\pi}_n \mathbf{1} = 1$

Discrete-time Markov chain

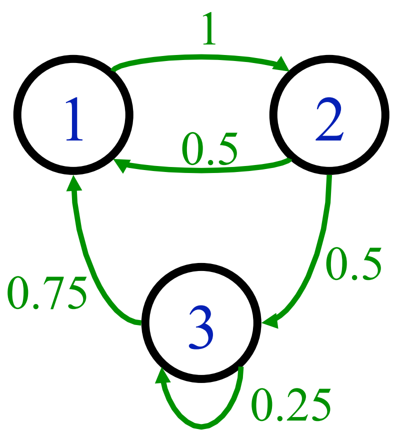
- More definitions
  - a DTMC is irreducible if *any* state can be reached from *any other* state
  - a state is aperiodic if the largest common divisor of all its cycles is 1 (e.g., states with loops are aperiodic)
  - a DTMC is aperiodic if *all* states are aperiodic
- Theorem

  if a DTMC is irreducible and aperiodic, and if there is a strictly positive solution to the set of equations

$$\begin{cases} \boldsymbol{\pi} = \boldsymbol{\pi}\,\mathbf{P} \\ \boldsymbol{\pi}\mathbf{1} = 1 \end{cases}$$

  then $\pi(i) = \lim_{n\to\infty} P(X_n = i)$ independent of the initial state

Simple example

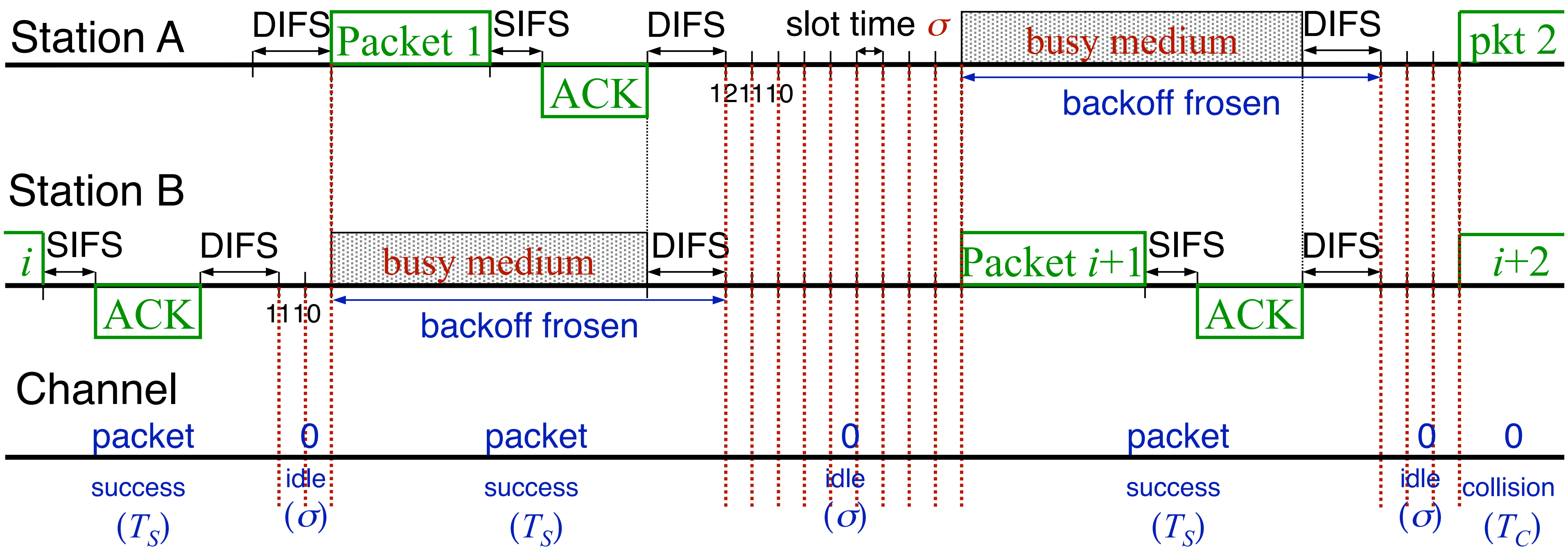


- It is a homogeneous DTMC because transitions depend only on current state
- It is irreducible
- It is aperiodic
- ➡ theorem applies to compute the stationary distribution π

$$\mathbf{P} = \begin{pmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0.75 & 0 & 0.25 \end{pmatrix}, \quad \boldsymbol{\pi} = \left(\pi(1), \pi(2), \pi(3)\right)$$

$$\begin{cases} \boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P} \\ \boldsymbol{\pi}\mathbf{1} = 1 \end{cases} \Rightarrow \quad \boldsymbol{\pi} = \left(0.375, 0.375, 0.25\right)$$

IEEE 802.11 Distributed Coordination Function



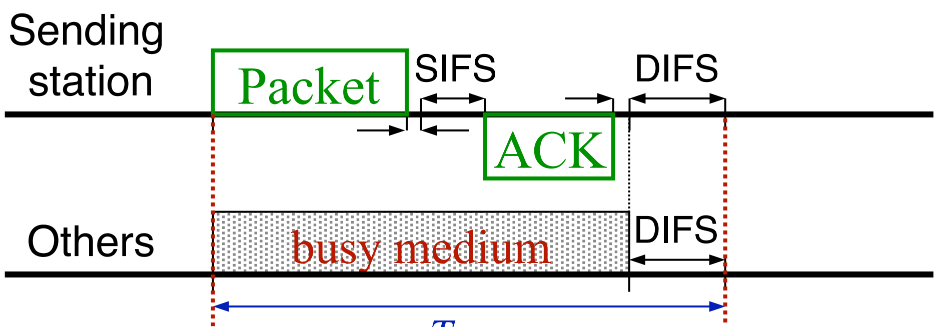
System throughput

$$S = \frac{P_{\text{success}}\, E[\text{data}] + \cancel{P_{\text{collision}}\, 0 + P_{\text{idle}}\, 0}}{P_{\text{success}}\, T_S + P_{\text{collision}}\, T_C + P_{\text{idle}}\, \sigma}$$

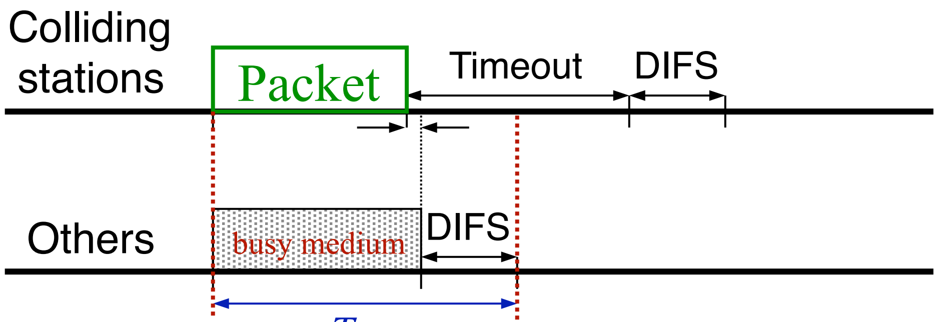
where $P_{\text{success}} + P_{\text{collision}} + P_{\text{idle}} = 1$

Computing T_S and T_C

Basic access mechanism

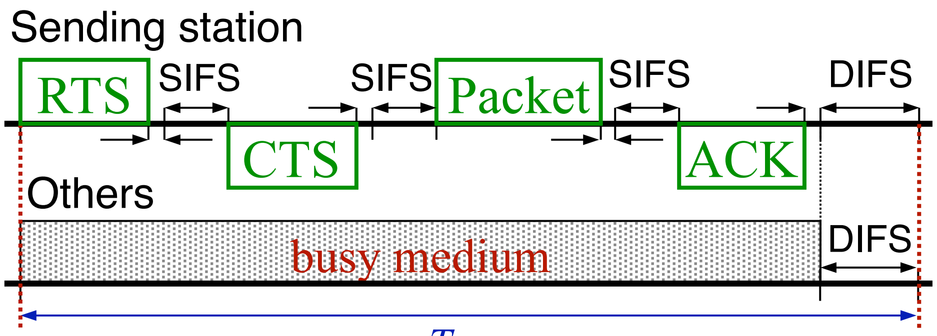


T_S =
Header + E[data] + δ + SIFS + ACK + δ + DIFS

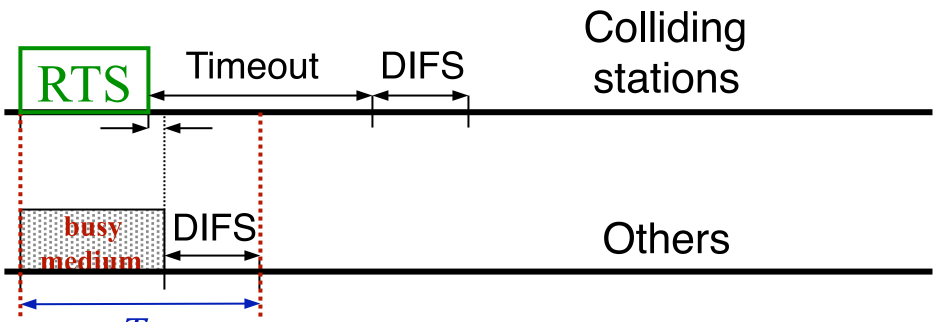


T_C =
Header + $E[\text{data}^*]$ + δ + DIFS

RTS/CTS access mechanism



T_S =
Header + E[data] + δ + SIFS + ACK + δ + DIFS
+ RTS + δ + SIFS + CTS + δ + SIFS



T_C =
RTS + δ + SIFS

Computing P_{idle}, P_{success} and $P_{\text{collision}}$

- Introduce
  - τ : prob. station transmits in random slot (unknown)
  - N : number of active sources
- We have
  - $P_{\text{idle}} = P$(0 station are transmitting) $= (1-\tau)^N$
  - $P_{\text{success}} = P$(only 1 station is transmitting) $= N\tau(1-\tau)^{N-1}$
  - $P_{\text{collision}} = P$(2 or more stations are transmitting)

$$= 1 - (1-\tau)^N - N\tau(1-\tau)^{N-1} = 1 - P_{\text{idle}} - P_{\text{success}}$$

$$S = \frac{E[\text{data}]}{T_S - T_C + \left(\underbrace{T_C/\sigma}_{T_C^*} - P_{\text{idle}}(T_C/\sigma - 1)\right)\dfrac{\sigma}{P_{\text{success}}}} = \boxed{\frac{E[\text{data}]}{T_S - T_C + \dfrac{T_C^* - (1-\tau)^N (T_C^* - 1)}{N\tau(1-\tau)^{N-1}}\sigma}}$$

$S = -$

Computing τ

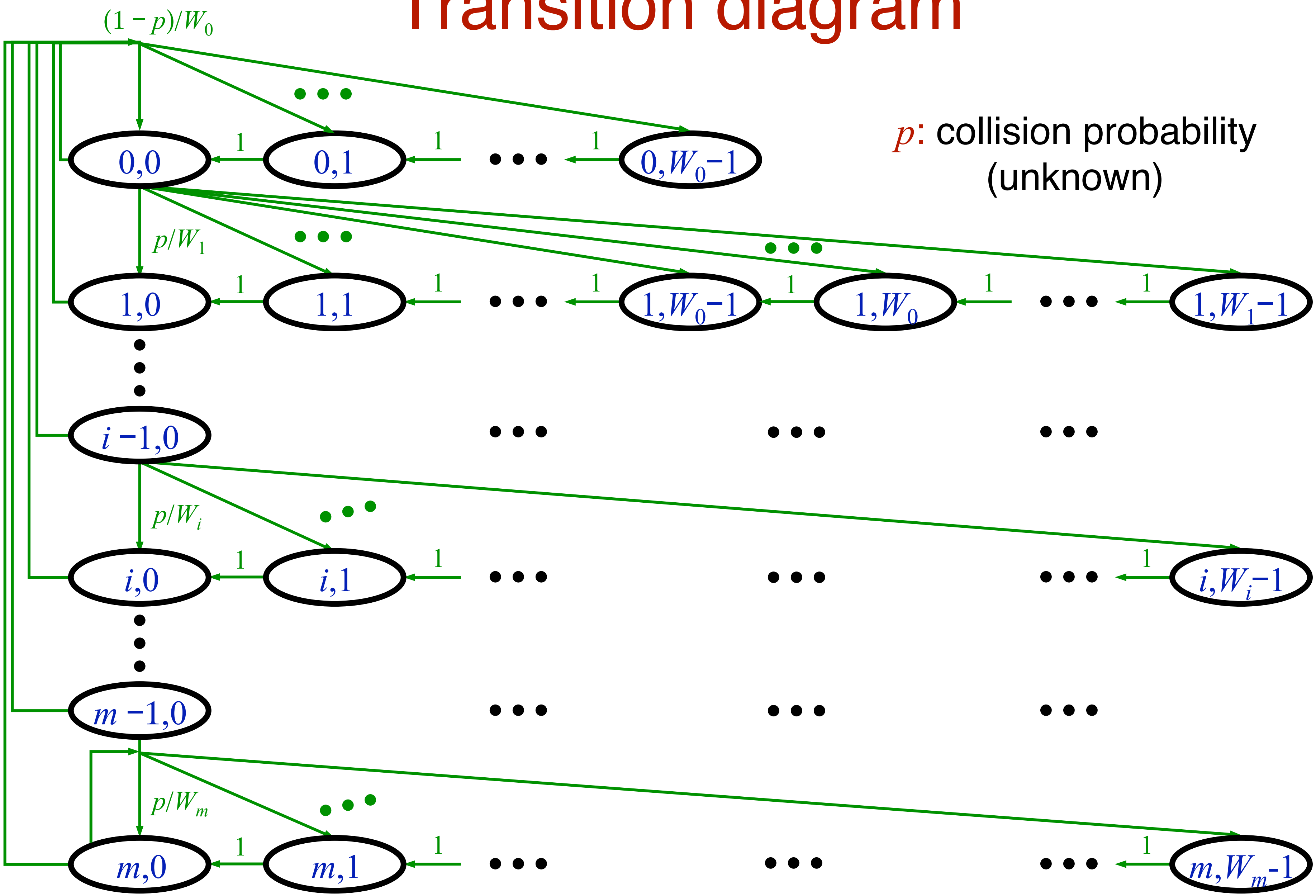
- Introduce
  - $X(n)$: exponential backoff stage at step n $(0 \le X(n) \le m)$
  - $Y(n)$: backoff value at step n $(0 \le Y(n) \le 2^{X(n)} W_0 - 1)$

$$\Rightarrow \quad \tau = P(Y = 0) = \sum_{i=0}^{m} P(X = i, Y = 0) = \sum_{i=0}^{m} b_{i,0}$$

- Problem: how to compute $b_{i,0}$?
- We have
  - Discrete time (time is slotted)
  - Future values of $(X(n), Y(n))$ depend only on current value
    - at every time-step, $Y(n)$ is decreased by 1
    - success $\Rightarrow$ $X(n+1)=0$, otherwise, $X(n+1) = \min\{m, X(n) + 1\}$

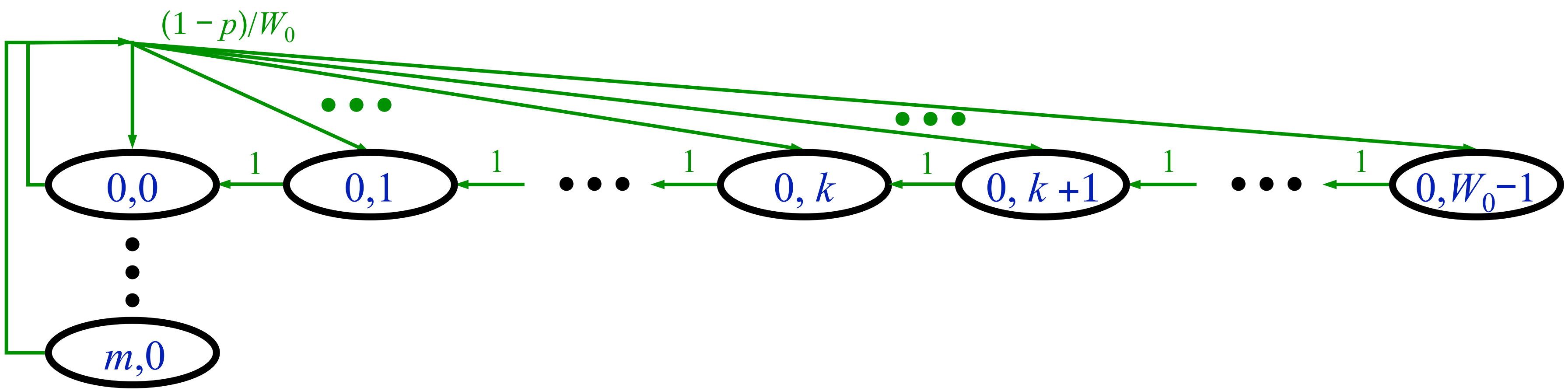
$\Rightarrow$ $(X(n), Y(n))$ homogeneous finite-space discrete-time 2-dimensional Markov chain

Transition diagram



p: collision probability (unknown)

Computing stationary distribution: $i = 0$

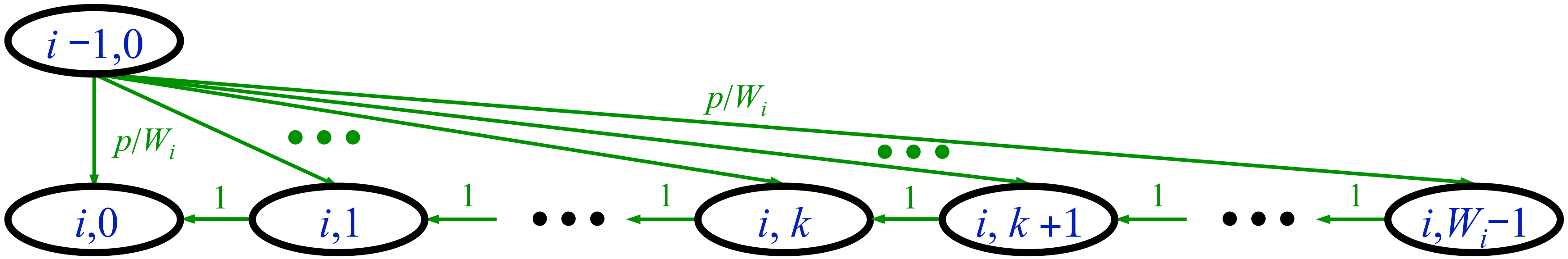


$$\begin{cases} b_{0,k} = b_{0,k+1} + \left((1-p)/W_0\right)\sum_0^m b_{i,0}, & \forall k \in [0, W_0 - 2] \\ b_{0,W_0-1} = \left((1-p)/W_0\right)\sum_{i=0}^m b_{i,0} \end{cases}$$

$$\Rightarrow \quad b_{0,k} = \sum_{j=1}^{W_0-k} \left((1-p)/W_0\right)\sum_0^m b_{i,0} = \frac{W_0 - k}{W_0}(1-p)\sum_0^m b_{i,0}, \forall k \in [0, W_0 - 1]$$

Observe that $b_{0,0} = (1-p)\sum_{i=0}^m b_{i,0}$ $\Rightarrow$ $b_{0,k} = \frac{W_0 - k}{W_0} b_{0,0}, \quad \forall k \in [0, W_0 - 1]$

Computing stationary distribution: $0 < i < m$

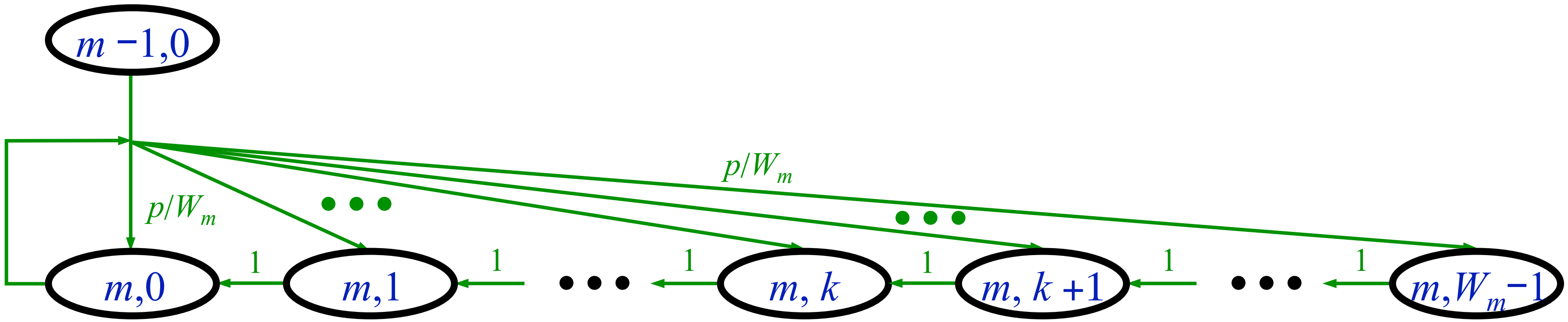


$$\begin{cases} b_{i,k} = b_{i,k+1} + (p/W_i) b_{i-1,0}, & \forall k \in [0, W_i - 2] \\ b_{i,W_i-1} = (p/W_i) b_{i-1,0} \end{cases}$$

$$\Rightarrow \quad b_{i,k} = \sum_{j=1}^{W_i - k} (p/W_i) b_{i-1,0} = \frac{W_i - k}{W_i} p\, b_{i-1,0}, \quad \forall k \in [0, W_i - 1]$$

Observe that $b_{i,0} = p\, b_{i-1,0}$ $\Rightarrow$ $b_{i,k} = \dfrac{W_i - k}{W_i} b_{i,0}, \quad \forall k \in [0, W_i - 1]$

Computing stationary distribution: $i = m$



$$\begin{cases} b_{m,k} = b_{m-1,k+1} + (p/W_m)(b_{m-1,0} + b_{m,0}) & \forall k \in [0, W_m - 2] \\ b_{m,W_m-1} = (p/W_m)(b_{m-1,0} + b_{m,0}) & \end{cases}$$

$$\Rightarrow \quad b_{m,k} = \sum_{j=1}^{W_m - k} (p/W_m)(b_{m-1,0} + b_{m,0}) = \frac{W_m - k}{W_m} p(b_{m-1,0} + b_{m,0}) \; \forall k \in [0, W_m - 1]$$

Observe that $\boxed{b_{m,0} = p(b_{m-1,0} + b_{m,0})} \Rightarrow \boxed{b_{m,k} = \frac{W_m - k}{W_m} b_{m,0}, \quad \forall k \in [0, W_m - 1]}$

Recapitulate

$$b_{i,k} = \frac{W_i - k}{W_i} b_{i,0}, \quad \forall k \in [0, W_i - 1] \text{ and } \forall i \in [0, m]$$

$$(1) \quad b_{0,0} = (1 - p) \sum_{i=0}^{m} b_{i,0}$$

$$(2) \quad b_{i,0} = p\, b_{i-1,0}, \quad \forall i \in [1, m-1] \quad \Rightarrow \quad b_{i,0} = p^i b_{0,0}, \quad \forall i \in [0, m-1]$$

$$(3) \quad b_{m,0} = p(b_{m-1,0} + b_{m,0}) \quad \Rightarrow \quad b_{m,0} = \frac{p}{1-p} b_{m-1,0} = \frac{p^m}{1-p} b_{0,0}$$

Next step to finish computing stationary distribution:

find $b_{0,0}$

Normalisation $\Rightarrow \quad \sum_{i=0}^{m} \sum_{k=0}^{W_i - 1} b_{i,k} = 1$

Finding $b_{0,0}$

$$\sum_{i=0}^{m}\sum_{k=0}^{W_i-1} b_{i,k} = 1 \quad \Leftrightarrow \quad 1 = \sum_{i=0}^{m} b_{i,0} \sum_{k=0}^{W_i-1} \frac{W_i - k}{W_i}$$

$$\Leftrightarrow \quad 1 = \sum_{i=0}^{m} b_{i,0} \frac{W_i + 1}{2} \qquad \text{(recall } W_i = 2^i W_0\text{)}$$

$$\Leftrightarrow \quad 1 = \sum_{i=0}^{m-1} p^i b_{0,0} \left(\frac{W_i + 1}{2} \right) + \frac{p^m}{1-p} b_{0,0} \left(\frac{W_m + 1}{2} \right)$$

$$\Leftrightarrow \quad 1 = \frac{b_{0,0}}{2} \left[W_0 \sum_{i=0}^{m-1} (2p)^i + \sum_{i=0}^{m-1} p^i + \frac{p^m}{1-p} \left(2^m W_0 + 1 \right) \right]$$

$$\Leftrightarrow \quad 1 = \frac{b_{0,0}}{2} \left[W_0 \left(\frac{1-(2p)^m}{1-2p} + \frac{(2p)^m}{1-p} \right) + \frac{1}{1-p} \right]$$

$$\Leftrightarrow \quad b_{0,0} = \frac{2(1-2p)(1-p)}{(1-2p)(W_0+1) + pW_0\left[1-(2p)^m\right]}$$

Performance measures

- Prob. station transmits in random slot is

$$\tau = P(Y=0) = \sum_{i=0}^{m} P(X=i, Y=0) = \sum_{i=0}^{m} b_{i,0}$$

$$= \left[\sum_{i=0}^{m-1} p^i + \frac{p^m}{1-p} \right] b_{0,0}$$

$$\tau = \frac{b_{0,0}}{1-p} = \frac{2(1-2p)}{(1-2p)(1+W_0) + pW_0\left[1-(2p)^m\right]}$$

- p is collision probability given that station is transmitting = P(at least 1 other station is transmitting)

$$p = 1-(1-\tau)^{N-1}$$

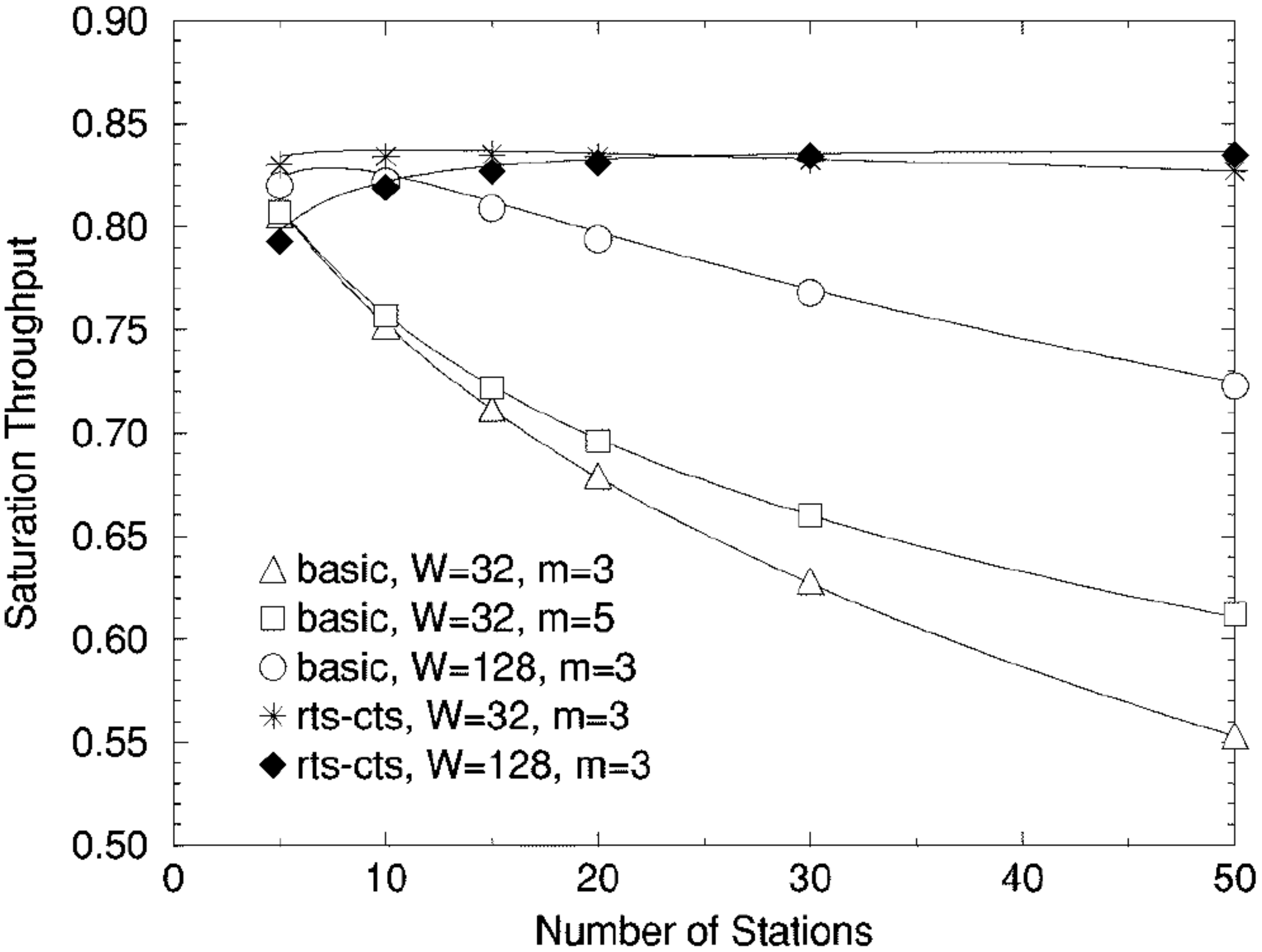
- $p = f(p) \quad \Rightarrow$ fixed-point equation

Summary

- We have computed
  - transmission probability $\tau = \dfrac{2(1-2p)}{(1-2p)(1+W_0)+pW_0\left[1-(2p)^m\right]}$
  - collision probability $p = 1-(1-\tau)^{N-1}$
  - normalized system throughput $S = \dfrac{E[\text{data}]}{T_S - T_C + \dfrac{T_C^* - (1-\tau)^N (T_C^* - 1)}{N\tau(1-\tau)^{N-1}}\sigma}$
- Reference:

  Giuseppe Bianchi "*Performance analysis of the IEEE 802.11 Distributed Coordination Function*", IEEE JSAC 18(3): 535-547, March 2000

Validation



0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
Saturation Throughput
0
10
20
30
40
50
Number of Stations
basic, W=32, m=3
basic, W=32, m=5
basic, W=128, m=3
rts-cts, W=32, m=3
rts-cts, W=128, m=3