Performance Evaluation Course
Master UBINET
Exam II part

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Ex. 1 — Solve the following games:

	A	B	C	D
A	5	1	4	-2
B	-5	2	1	4

	A	B
A	(0,1)	(1,2)
B	(1,3)	(0,1)

Ex. 2 — FTP accelerator.
There are 2 users that want to download a content from an FTP server with uploading bandwidth $C = 1$. They can both use an FTP accelerator that opens up to a maximum of N TCP connections in parallel in order to speed-up the download. The users can decide the number $n = 0, 1, 2 \cdot N$ of connections to open in parallel. Assume that the open TCP connections equally share the available uploading bandwidth.

1. Consider $N = 2$ and model user interactions as a game. Study this game, in particular
   - determine if it is zero-sum or not,
   - determine equilibria in pure and mixed strategies and Pareto optimal outcomes.

   For N larger than 2, what do you expect?
2. Consider now that the utility of a user is equal to its downloading rate minus a cost proportional, according to a parameter α, to the number of open connections. Discuss if and how the pure strategy Nash Equilibria change, for $N = 2$, when α increases from 0 to 1. Say also if they are Pareto optimal.
3. Imagine that the one-shot game defined above (with $N = 2$ and $\alpha = 0.1$) is the basic stage of a repeated game with discount factor δ.
   - In comparison to the single-stage game, can better outcomes arise as equilibria if the repeated game has finite horizon? and if it has infinite horizon?
   - Propose a trigger strategy that would lead to a subgame Nash Equilibrium that is also Pareto optimal. For which values of the discount factor would this happen?

Ex. 3 — Hawks, Doves, Patient Doves
Extend the Hawk-Dove game considering a variety of patient doves, that are willing to afford a longer posturing phase.
Here we recap the game. Members of a specie engage in repeated random conflicts over some resource, whose value is 50 fitness points. There are three possible behaviours:

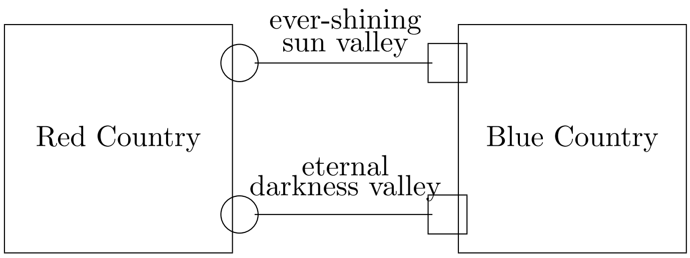
hawk: fights for resources until the adversary does not escape or it is injured (-100);

dove: starts a merely symbolic conflict, posturing and threatening but not fighting. This symbolic conflict costs 10 points;

patient dove: as a normal dove starts a merely symbolic conflict, but it is willing to carry on it for a longer time up to a cost of 20 points (but for example will only afford a cost of 10 points when competing with a dove).

Determine Evolutionary Stable Strategies.

Ex. 4 — Fortresses.

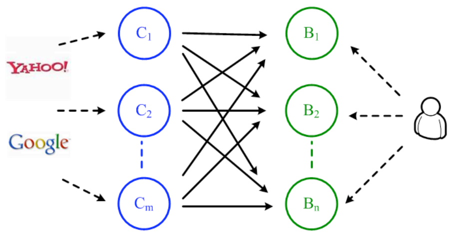


In a far, far, far away land, two countries, Red and Blue have not very pacific relations. Red's army has 2 battalions, Blue's army has 3 battalions. Two valleys allow communication among the two countries: the ever-shining sun valley and the eternal darkness valley. In each valley a Red fortress faces a Blue one.
Red has decided to deploy its battalions at the two fortresses. A battalion cannot be split. **After** it does so, Blue will also deploy its battalions. **Then** Red may decide to attack from one of its fortresses Blue's fortress in the same valley. Blue will only defend if attacked. An attack can only be carried from a fortress where there is at least one battalion to the other fortress in the same valley. The attack will involve all the battalions in the fortress. The attackers will conquer the fortress only if they have strict numerical superiority and in this case they will not suffer any loss. In all the other cases the defenders will win and the attacking battalions will be destroyed. Model this conflict as a zero-sum game tree, assuming that the value of conquering a fortress is $+5$ and the loss of a battalion is -1. Note that Blue's strategy should specify how to divide the battalions, while Red strategy

should specify: 1) how to divide the battalions, 2) whether to attack or not, and 3) which fortress to attack (if any).

1. Produce the tree and study the equilibria if every country knows how the other one has deployed its battalions.
2. Produce the tree and study the equilibria if no country has any information about how the other one has deployed its battalions.
3. What would happen if Blue knows Red's battalions positions, Red is aware of it but it does not know Blue's battalions positions? What if Blue knows Red's battalions positions but Red is not aware of it and does not know Blue's battalions positions?

Ex. 5 — ISP fair share.
Internet Service Providers (ISPs) require the cooperation of other ISPs to provide Internet services to their customers. Consider the two-sided market model as shown in the figure below. On the left side, content providers, e.g. Google and Yahoo, provide information through m content ISPs denoted as C_i ; on the right side, end-customers access Internet and download information from n eyeball ISPs denoted as B_j. Suppose the total profit generated by serving both content providers and end customers is V. This profit is generated only if there is at least one content ISP and one eyeball ISP.



1. Model the interaction among the ISPs as a coalition game. Is this game super-additive in general?
2. Consider the case of $m = 1$ and $n = 2$. Determine the core and the Shapley value of the game. How do you expect the Shapley value to change if n increases? How if m increases?
3. Derive the Shapley value for generic n and m. Suggestion: it is much simpler to consider the aggregated imputation of content ISPs -or of eyeball ISPs- and then apply symmetry property. (Do not get stuck on this question.)

Ex. 6 — Consider 2-hop routing and epidemic routing with a timer based recovery process. In particular, assume that whenever a relay node receives a copy of the message, it starts a timer. When the time expires, the copy is erased. The node does not keep track of the erased messages so that it can later receive another copy of the same message. The source does not use the timer. Assume that timer durations are independent and identically distributed exponential random variables with expected value $1/\rho$.

1. Modify the Markovian model presented in the course in order to include also the timer.
2. Derive the corresponding fluid model for the number of infected nodes in the system under Kurtz's limit. What does this model predict for the number of infected nodes in the system for large time instants?

Ex. 7 — Battleship.
Two players are involved in an exciting battleship. Player 1 has a 2 by 3 grid and is going to hide in it one cruiser of length equal to 2 squares. The other one has one trial to shoot at its opponent. The figure shows two possible positions for the cruiser.
Model this game as a zero sum game, where Player 1 incurs a cost 1 equal to 1 if his cruiser is hit, 0 otherwise. Determine the equilibria of the game.