Game Theory: introduction and applications to computer networks

Lecture 2: two-person non zero-sum games (part A)

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Slides are based on a previous course
with D. Figueiredo (UFRJ) and H. Zhang (Suffolk University)

Outline

- Two-person zero-sum games
  - Matrix games
    - Pure strategy equilibria (dominance and saddle points), ch 2
    - Mixed strategy equilibria, ch 3
  - Game trees, ch 7
  - About utility, ch 9
- Two-person non-zero-sum games
  - Nash equilibria…
    - …And its limits (equivalence, interchangeability, Prisoner's dilemma), ch. 11 and 12
  - Strategic games, ch. 14
  - Subgame Perfect Nash Equilibria (not in the book)
  - Repeated Games, partially in ch. 12
  - Evolutionary games, ch. 15
- N-persons games

References

- Main one:
  - Straffin, Game Theory and Strategy, The mathematical association of America
- Subjects not covered by Straffin:
  - Osborne and Rubinstein, A course in game theory, MIT Press

Two-person Non-zero Sum Games

- Players are not strictly opposed
  - payoff sum is non-zero

Player 2

Player 1		A	B
	A	3, 4	2, 0
	B	5, 1	-1, 2

- Situations where interest is not directly opposed
  - players could cooperate
  - communication may play an important role
    - for the moment assume no communication is possible

What do we keep from zero-sum games?

- Dominance
- Movement diagram
  - pay attention to which payoffs have to be considered to decide movements

		Player 2 A	Player 2 B
Player 1	A	5, 4	2, 0
	B	3, 1	-1, 2

- Enough to determine pure strategies equilibria
  - but still there are some differences (see after)

What can we keep from zero-sum games?

- As in zero-sum games, pure strategies equilibria do not always exist…

		Player 2	
		A	B
Player 1	A	5, 0	-1, 4
	B	3, 2	2, 1

- …but we can find mixed strategies equilibria

Mixed strategies equilibria

- Same idea of equilibrium
  - each player plays a mixed strategy (*equalizing strategy*), that equalizes the opponent payoffs
  - how to calculate it?

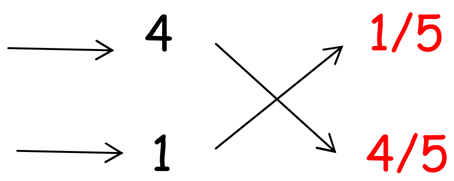
		Colin A	Colin B
Rose	A	5, 0	-1, 4
	B	3, 2	2, 1

Mixed strategies equilibria

- Same idea of equilibrium
  - each player plays a mixed strategy, that equalizes the opponent payoffs
  - how to calculate it?

		Colin A	Colin B
Rose	A	-0	-4
	B	-2	-1

Rose considers *Colin's game*



Mixed strategies equilibria

- Same idea of equilibrium
  - each player plays a mixed strategy, that equalizes the opponent payoffs
  - how to calculate it?

		Colin A	Colin B
Rose	A	5	-1
	B	3	2
		3/5	2/5

Colin considers *Rose's game*

Mixed strategies equilibria

- Same idea of equilibrium
  - each player plays a mixed strategy, that equalizes the opponent payoffs
  - how to calculate it?

		Colin	
		A	B
Rose	A	5, 0	-1, 4
	B	3, 2	2, 1

Rose playing (1/5,4/5)
Colin playing (3/5,2/5)
is an equilibrium

Rose gains 13/5
Colin gains 8/5

Good news: Nash's theorem [1950]

- Every two-person games has at least one equilibrium either in pure strategies or in mixed strategies
  - Proved using fixed point theorem
  - generalized to N person game
- This equilibrium concept called Nash equilibrium in his honor
  - A vector of strategies (a profile) is a Nash Equilibrium (NE) if no player can unilaterally change its strategy and increase its payoff

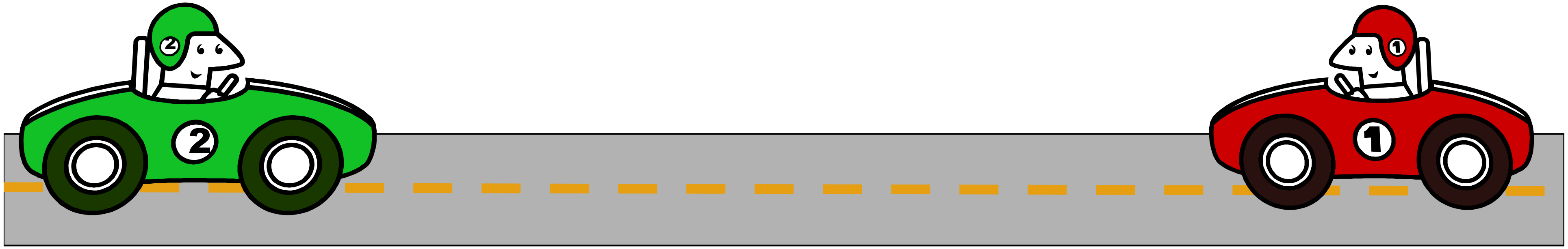
An useful property

- Given a finite game, a profile is a mixed NE of the game if and only if for every player i, every pure strategy used by i with non-null probability is a best response to other players mixed strategies in the profile
  - see Osborne and Rubinstein, *A course in game theory*, Lemma 33.2

Bad news: what do we lose?

- equivalence
- interchangeability
- identity of equalizing strategies with prudential strategies
- main cause
  - at equilibrium every player is considering the opponent's payoffs ignoring its payoffs.
- New problematic aspect
  - group rationality versus individual rationality (cooperation versus competition)
  - absent in zero-sum games
- we lose the idea of **the** solution

Game of Chicken



- Game of Chicken (aka. Hawk-Dove Game)
  - driver who swerves looses

Driver 1 \ Driver 2	swerve	stay
swerve	0, 0	-1, 5
stay	5, -1	-10, -10

Drivers want to do opposite of one another

Two equilibria:
not equivalent
not interchangeable!

• playing an equilibrium strategy does not lead to equilibrium

Prudential strategies

- Each player tries to minimize its maximum loss (then it plays in its own game)

		Colin A	Colin B
Rose	A	5, 0	-1, 4
	B	3, 2	2, 1

Prudential strategies

- Rose assumes that Colin would like to minimize her gain
- Rose plays in Rose's game
- Saddle point in BB
- B is Rose's prudential strategy and guarantees to Rose at least 2 (Rose's *security level*)

		Colin	
		A	B
Rose	A	5	-1
	B	3	2

Prudential strategies

- Colin assumes that Rose would like to minimize his gain (maximize his loss)
- Colin plays in Colin's game
- mixed strategy equilibrium,
- (3/5,2/5) is Colin's prudential strategy and guarantees Colin a gain not smaller than 8/5

		Colin	
		A	B
Rose	A	0	-4
	B	-2	-1

Prudential strategies

- Prudential strategies
  - Rose plays B, Colin plays *A* w. prob. 3/5, B w. 2/5
  - Rose gains 13/5 (>2), Colin gains 8/5
- Is it stable?
  - No, if Colin thinks that Rose plays B, he would be better off by playing *A* (Colin's *counter-prudential strategy*)

		Colin	
		A	B
Rose	*A*	5, 0	-1, 4
	B	3, 2	2, 1

Prudential strategies

- are not the solution neither:
  - do not lead to equilibria
  - do not solve the group rationality versus individual rationality conflict
- dual basic problem:
  - look at your payoff, ignoring the payoffs of the opponents

The Prisoner's Dilemma

- One of the most studied and used games
  - proposed in 1950
- Two suspects arrested for joint crime
  - each suspect when interrogated separately, has option to confess

		Suspect 2: NC	Suspect 2: C
Suspect 1	NC	2, 2	10, 1
	C	1, 10	5, 5

payoff is years in jail
(**smaller is better**)

better outcome

single NE

Pareto Optimal

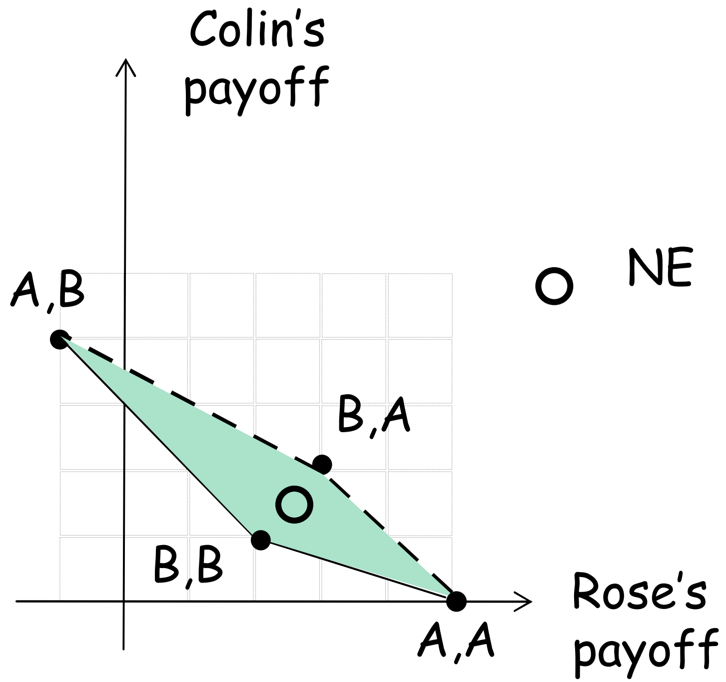
		Suspect 2 NC	Suspect 2 C
Suspect 1	NC	2, 2	10, 1
	C	1, 10	5, 5

Highlighted: Pareto Optimal (2, 2; 10, 1; 1, 10)

- Def: outcome o* is Pareto Optimal if no other outcome would give to all the players a payoff not smaller and a payoff higher to at least one of them
- Pareto Principle: to be acceptable as a solution of a game, an outcome should be Pareto Optimal
  - the NE of the Prisoner's dilemma is not!
- Conflict between group rationality (Pareto principle) and individual rationality (dominance principle)

Payoff polygon

Rose		Colin A	Colin B
	A	5, 0	-1, 4
	B	3, 2	2, 1



- All the points in the convex hull of the pure strategy payoffs correspond to payoffs obtainable by mixed strategies
- The north-east boundary contains the Pareto optimal points

Exercises

- Find NE and Pareto optimal outcomes:

	NC	C
NC	2, 2	10, 1
C	1, 10	5, 5

	A	B
A	2, 3	3, 2
B	1, 0	0, 1

	swerve	stay
swerve	0, 0	-1, 5
stay	5, -1	-10, -10

	A	B
A	2, 4	1, 0
B	3, 1	0, 4